Augmented Matrix: is a matrix that contains the coefficient matrix & the matrix of constants.

$$x - 2y = 7 \rightarrow Eq1$$
$$-3x + 5y = -4 \rightarrow Eq2$$

$$\begin{array}{cc} \text{coefficient} & \text{constants} \\ \begin{array}{cc} x & y \end{array} & \\ \left[\begin{array}{cc|c} 1 & -2 & 7 \\ -3 & 5 & -4 \end{array} \right] \end{array}$$

Write an augmented matrix that could be used to solve each system of equations.

Ex. 1:

$$x - y = -3$$
$$x + 3y = 5$$

$$\left[\begin{array}{cc|c} 1 & -1 & -3 \\ 1 & 3 & 5 \end{array} \right]$$

Ex. 2:

$$2g + 3h = 8$$
$$-4g - 7h = -5$$

$$\left[\begin{array}{cc|c} 2 & 3 & 8 \\ -4 & -7 & -5 \end{array} \right]$$

Ex. 3:

$$3a - 5b + 2c = 9$$
$$4a + 7b + c = 3$$
$$2a - c = 12$$

$$\left[\begin{array}{ccc|c} 3 & -5 & 2 & 9 \\ 4 & 7 & 1 & 3 \\ 2 & 0 & -1 & 12 \end{array} \right]$$

STEPS FOR SOLVING A SYSTEM USING AUGMENTED MATRICES:

Goal: the coefficient matrix $\rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

① Multiply Rows by #

② Add Rows together

* Exchange Row Locations

start $\begin{bmatrix} 1 & 0 & | & - \\ 0 & 1 & | & - \end{bmatrix}$

Ex. 4: Solve this system of equations using an augmented matrix.

$2x + y = 19$

$x - y = 11$

$\begin{bmatrix} 2 & 1 & | & 19 \\ 1 & -1 & | & 11 \end{bmatrix} = \overset{(-2)}{\begin{bmatrix} \overset{-2}{\underset{1}{\frac{1}{2}}} & \overset{2}{-1} & | & \overset{-22}{11} \\ 2 & 1 & | & 19 \end{bmatrix}}$

To get a 0:

① Multiply a Row (Fake Row)

② Add 2 Rows

$\div 3 \begin{bmatrix} \underline{1} & -1 & | & 11 \\ \underline{0} & 3 & | & -3 \end{bmatrix} = \begin{bmatrix} \underline{\frac{1}{}} & -1 & | & 11 \\ \underline{0} & 1 & | & -1 \end{bmatrix}$

To get a 1:

Multiply / Divide a Row

$\begin{bmatrix} 1 & 0 & | & 10 \\ 0 & 1 & | & -1 \end{bmatrix}$

$X = 10 \qquad y = -1$

Solve each system of equations using an augmented matrix.

Ex. 5:

$x - y = -3$

$x + 3y = 5$

$(-1)\begin{bmatrix} -1 & 1 & | & 3 \\ 1 & -1 & | & -3 \\ 1 & 3 & | & 5 \end{bmatrix} \overset{?}{\div 4} \begin{bmatrix} 1 & -1 & | & -3 \\ 0 & 4 & | & 8 \end{bmatrix}$

$\begin{bmatrix} 1 & -1 & | & -3 \\ 0 & 1 & | & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & | & -1 \\ 0 & 1 & | & 2 \end{bmatrix}$

$x = -1$

$y = 2$

Ex. 6:

$x - 2y = 1$

$x + 5y = 22$

$(-1)\begin{bmatrix} -1 & 2 & | & -1 \\ 1 & -2 & | & 1 \\ 1 & 5 & | & 22 \end{bmatrix} = \div 7 \begin{bmatrix} 1 & -2 & | & 1 \\ 0 & 7 & | & 21 \end{bmatrix}$

$(2)\begin{bmatrix} 1 & -2 & | & 1 \\ 0 & 1 & | & 3 \\ 0 & 2 & & 6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & | & 7 \\ 0 & 1 & | & 3 \end{bmatrix}$

$x = 7 \qquad y = 3$

Solve each system of equations using an augmented matrix.

Ex. 7:

$3x + 3y = -9$

$5x + 3y = -11$

$\div 3 \begin{bmatrix} 3 & 3 & | & -9 \\ 5 & 3 & | & -11 \end{bmatrix} = (-5)\begin{bmatrix} 1 & 1 & | & -3 \\ 5 & 3 & | & -11 \end{bmatrix}$

$-5 \quad -5 \quad 15$

$\div -2 \begin{bmatrix} 1 & 1 & | & -3 \\ 0 & -2 & | & 4 \end{bmatrix} = (-1)\begin{bmatrix} 1 & 1 & | & -3 \\ 0 & 1 & | & -2 \end{bmatrix}$

$0 \quad -1 \quad 2$

$\begin{bmatrix} 1 & 0 & | & -1 \\ 0 & 1 & | & -2 \end{bmatrix}$

Ex. 8:

$-x - 7y = 14$

$-4x - 14y = 28$