Identity Matrix:

2x2

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

3x3

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Inverses:

$$A \cdot B = I$$

The Inverse of a 2x2 Matrix

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Ex. 1: Find the inverse of $A = \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}$

$|A| = 6 - 4 = 2$

① Find the determinant

$$\frac{1}{2}\begin{bmatrix} 2 & -1 \\ -4 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -1/2 \\ -2 & 3/2 \end{bmatrix}$$

Ex. 2: Solve the following equations.

$\frac{1}{5}\cdot 5 \quad \frac{1}{5}\cdot -15$

$$\frac{5x}{5} = \frac{-15}{5}$$

$x = -3$

$$\frac{2}{3}\cdot\frac{3}{2}x = \frac{27}{1}\cdot\frac{2}{3}$$

$x = 6$

Matrix Equations:

When multiplying by the inverse, the inverse matrix needs to be on the LEFT.

Ex. 3: Solve the matrix equation $AX = B$ for the 2×2 matrix X.

$A^{-1}AX = A^{-1}B$

$IX = A^{-1}B$

① Find the Inverse of A.

② Multiply $A^{-1}B = X$ on LEFT

$$\overbrace{\begin{bmatrix} 4 & -1 \\ -3 & 1 \end{bmatrix}}^{A^{-1}\ A} X = \overbrace{\begin{bmatrix} 8 & -5 \\ -6 & 3 \end{bmatrix}}^{A^{-1}\ B}$$

$$\begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 8 & -5 \\ -6 & 3 \end{bmatrix} = \begin{bmatrix} 8-6 & -5+3 \\ 24-24 & -15+12 \end{bmatrix}$$

$|A| = 4-3 = 1$

$$A^{-1} = \frac{1}{1}\begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix} \qquad A^{-1} = \begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix}$$

$$X = \begin{bmatrix} 2 & -2 \\ 0 & -3 \end{bmatrix}$$

Ex. 4: Solve the following matrix equation.

$$\begin{bmatrix} 5 & 3 \\ -2 & 4 \end{bmatrix} X + \begin{bmatrix} -3 & -10 \\ 1 & 15 \end{bmatrix} = \begin{bmatrix} 4 & 11 \\ 19 & 43 \end{bmatrix}$$

$$- \begin{bmatrix} -3 & -10 \\ 1 & 15 \end{bmatrix}$$

$$A^{-1}AX = A^{-1}B$$

$$\begin{bmatrix} 5 & 3 \\ -2 & 4 \end{bmatrix} X = \begin{bmatrix} 7 & 21 \\ 18 & 28 \end{bmatrix}$$

$|A| = 20 - -6 = 26$

$$A^{-1} = \frac{1}{26}\begin{bmatrix} 4 & -3 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} \frac{4}{26} & \frac{-3}{26} \\ \frac{2}{26} & \frac{5}{26} \end{bmatrix} \begin{bmatrix} 7 & 21 \\ 18 & 28 \end{bmatrix} = \begin{bmatrix} \frac{28}{26} - \frac{54}{26} & \frac{84}{26} - \frac{84}{26} \\ \frac{14}{26} + \frac{90}{26} & \frac{42}{26} + \frac{140}{26} \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 \\ 4 & 7 \end{bmatrix}$$

$\frac{-26}{26}$ $\frac{182}{26}$

How do we tell if two matrices are inverses of each other?

Multiply & see if they equal the Identity