Examples 9-3
Multiplying Matrices

<u>Multiplying Two Matrices</u>: The product of 2 matrices is only defined (is possible) when the # of COLUMNS in the 1st matrix is equal to the # of ROWS in the 2nd matrix.

<u>Dimensions</u>

$A \rightarrow m \times n$

$B \rightarrow n \times p$

$$\underset{m \times \boxed{n}\ \boxed{n} \times p}{A \cdot B} = \underset{m \times p}{AB}$$

Ex. 1: State whether the product ***AB*** is defined. If so, give the dimensions of ***AB***.

a. $\boldsymbol{A: 2\times 3,\ \ B: 3\times 4}$

$$\underset{2\times \boxed{3 \quad 3}\times 4}{A \cdot B} = \underset{2\times 4}{AB}$$

BA

3x4 2x3

Not defined

b. $\boldsymbol{A: 3\times 2,\ \ B: 3\times 4}$

A B

3x2 3x4

Not defined

Finding the Product of Two Matrices:

If it is defined:

Multiply the corresponding entries of the ROWS of the 1st Matrix to the COLUMNS of the 2nd Matrix, then add.

Ex. 2: Find AB if $A = \begin{array}{c} \text{ROWS} \\ \begin{array}{c} R1 \\ R2 \\ \\ \end{array} \begin{bmatrix} -2 & 3 \\ 1 & -4 \\ 6 & 0 \end{bmatrix} \end{array}$ and $B = \begin{array}{c} \text{COLUMNS} \\ \begin{array}{cc} C1 & C2 \end{array} \\ \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix} \end{array}$

$3\times\boxed{2\ 2}\times 2$

$AB \rightarrow 3 \times 2$

$$\begin{bmatrix} \underline{R1C1} & \underline{R1C2} \\ \underline{R2C1} & \underline{R2C2} \\ \underline{R3C1} & \underline{R3C2} \end{bmatrix}$$

$$\begin{array}{c} R1C1 \\ R2C1 \\ R3C1 \end{array} \begin{bmatrix} \frac{(-2)(-1) + (3)(-2)}{2 + -6} & \frac{(-2)(3) + (3)(4)}{-6 + 12} \\ \underline{-1 + 8} & \underline{3 + -16} \\ \underline{-6 + 0} & \underline{18 + 0} \end{bmatrix} \begin{array}{c} R1C2 \\ R2C2 \\ R3C2 \end{array}$$

$$\begin{bmatrix} -4 & 6 \\ 7 & -13 \\ -6 & 18 \end{bmatrix}$$

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Ex. 3: If $\boldsymbol{A} = \begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix}$ and $\boldsymbol{B} = \begin{bmatrix} 1 & -4 \\ 2 & 1 \end{bmatrix}$, find each product.

$2 \times \boxed{2 \quad\quad 2} \times 2$

a. ***AB***

R1C1 $\begin{bmatrix} 3+4 & -12+2 \\ -1+0 & 4+0 \end{bmatrix} = \begin{bmatrix} 7 & -10 \\ -1 & 4 \end{bmatrix}$

b. ***BA***

$\begin{array}{ccc} B & \cdot & A \\ \begin{bmatrix} 1 & -4 \\ 2 & 1 \end{bmatrix} & & \begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix} \end{array}$

$\begin{bmatrix} 3+4 & 2+0 \\ 6+-1 & 4+0 \end{bmatrix} = \begin{bmatrix} 7 & 2 \\ 5 & 4 \end{bmatrix}$

Multiply the following matrices, if possible.

Ex. 4: $\begin{bmatrix} \mathbf{2} & \mathbf{-1} & \mathbf{4} \\ \mathbf{-1} & \mathbf{3} & \mathbf{0} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{-4} & \mathbf{5} \\ \mathbf{0} & \mathbf{2} \\ \mathbf{-1} & \mathbf{7} \end{bmatrix}$

$2 \times 3 \quad\quad 3 \times 2$

R1C1 $\begin{bmatrix} -8+-4 & 10+-2+28 \\ 4+0+0 & -5+6+0 \end{bmatrix}$

$\begin{bmatrix} -12 & 36 \\ 4 & 1 \end{bmatrix}$

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Ex. 5: $\begin{bmatrix} -3 & 4 \\ 0 & 6 \end{bmatrix} \cdot \begin{bmatrix} 0 & 9 \\ -1 & 8 \end{bmatrix}$ 2x2

Ex. 6: $\begin{bmatrix} 2 & 1 \\ -9 & 0 \\ 3 & 5 \end{bmatrix} \cdot \begin{bmatrix} -5 & 6 & 7 & 3 \end{bmatrix}$

3x2 1x4

Not defined

Ex. 7: $\begin{bmatrix} \boxed{-3} \\ 9 \\ 8 \end{bmatrix} \cdot \begin{bmatrix} \boxed{7} & -8 & 1 \end{bmatrix}$

3x1 1x3

R1C1 $\begin{bmatrix} -21 & 24 & -3 \\ \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{bmatrix}$

Ex. 8: $\begin{bmatrix} 2 & -5 \\ 6 & 7 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 2 & -5 \\ 6 & 7 \end{bmatrix}$