Growth & Decay

Exponential Functions: $y = a \cdot b^x$

$a \rightarrow$ y-int
initial value
$b \rightarrow$ common ratio

Exponential Growth Functions:

$a > 0$ & $b > 1$ growth function

$$y = a(\underbrace{1+r})^t$$

Growth Factor: b

$b = 1 + r$ 100% + rate

Rate: r % of increase / decrease, ALWAYS is written as a decimal.

50% = 0.5
123% = 1.23
8% = 0.08
0.7% = 0.007

Ex. 1: In 1996, there were 2573 computer viruses and other computer security incidents. The number of incidents increased by about 92% each year. 1996 $\rightarrow$ $t = 0$

a) Write an exponential model (equation) giving the number n of incidents t years after 1996.

$$n = 2573(1 + .92)^t$$

$$n = 2573(1.92)^t$$

$$n = 2573(1.92)^{t-1996}$$

b) Use your equation to determine how many incidents were there in 2003?

1996 → 2003
$t = 0$ $t = 7$

$$\begin{array}{r} 2003 \\ -1996 \\ \hline 7 \end{array}$$

$$n = 2573(1.92)^7$$

$n = 247{,}485$ incidents

Exponential Decay Functions:

$a > 0$

$0 < b < 1$

$$y = a(\underbrace{1-r}_{})^t$$

Decay Factor: (b)

$b = 1 - r$ 100% − rate

Ex. 2: A new snowmobile costs $4200. The value of the snowmobile decreases by 10% each year.

a) Write an exponential model (equation) giving the snowmobile's value *y* (in dollars) after *t* years.

$y = 4200(1-0.1)^t$ $y = 4200(0.9)^t$

b) Estimate the value of the snowmobile after 3 years.

$y = 4200(0.9)^3$

$y = \$3061.80$

Steps for writing an exponential growth or decay function:

① Decide whether it is growth or decay

② Identify the initial value (a) & the rate (r) (decimal)

③ Plug in a & r

Ex. 3: A ball is dropped from 8 feet above the ground. Each bounce is $\boxed{80\%}$ of its previous height.

Decay Factor

$(1-.20)$ (0.8)

% of dec

$r = 20\%$

a) Write an exponential model giving the ball's height h (in feet) after each bounce n.

$$h = 8(0.8)^n$$

b) How high will the ball be after 5 bounces

$h = 8(0.8)^5$ $h = 2.62$ feet

Ex. 4: The algae population begins with 1000 in 2009. The algae population in a pond increases by 3% each year.

a) Write an exponential model giving the number n of algae t years after 2009.

$n = 1000(1+0.03)^t$

$n = 1000(1.03)^t$

b) How big will the population be in 2018?

$2018 - 2009 = 9$

$n = 1000(1.03)^9$

$n = 1305$ algae

Ex. 5: The air slowly kills bacteria found on a surface. The bacteria population began at 3000 and decreases by 5% each minute.

a) Write an exponential model giving the population p of bacteria after m minutes.

$p = 3000(1-0.05)^m$

$p = 3000(0.95)^m$

b) What will be the bacteria population after 21 minutes?

$p = 3000(0.95)^{21}$

$p = 1022$ bacteria

Ex. 6: A number of bacteria, $f(t)$, at any time t, in hours, can be estimated using the function $f(t) = 3000(1.24)^t$.

a) What was the initial size of the bacteria colony?

3000 bacteria

b) What is the growth/decay factor for the situation?

1.24

c) Is the bacteria population exponentially decaying or growing? How do you know?

$b > 1$ growth

$0 < b < 1$ decay

d) At what rate is the population changing?

Factor 1.24

$1 + r = 1.24$ -1

$r = 0.24$

24%

Factor 0.87

100% − rate = 87%

100%
−87%
13%

Factor 3.42

$1 + r = 3.42$
-1 -1

$r = 2.42$

242%